

TEST OF THE FLRW METRIC AND CURVATURE WITH STRONG LENS TIME DELAYS

KAI LIAO¹, ZHENGXIANG LI², GUO-JIAN WANG², XI-LONG FAN³

¹ School of Science, Wuhan University of Technology, Wuhan 430070, China.

² Department of Astronomy, Beijing Normal University, Beijing 100875, China.

³ Department of Physics and Mechanical and Electrical Engineering, Hubei University of Education, Wuhan 430205, China.

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ABSTRACT

We present a new model-independent strategy for testing the Friedmann-Lemaître-Robertson-Walker metric and constraining cosmic curvature, based on future time delay measurements of strongly lensed quasar-elliptical galaxy systems from the Large Synoptic Survey Telescope and supernova observations from the Dark Energy Survey. The test only relies on geometric optics. It is independent of the energy contents of the universe and the validity of the Einstein equation on cosmological scales. The study comprises two levels: testing the FLRW metric through the Distance Sum Rule and determining/constraining cosmic curvature. We propose an effective and efficient (redshift) evolution model for performing the former test, which allows us to concretely specify the violation criterion for the FLRW Distance Sum Rule. If the FLRW metric is consistent with the observations, then, on the second level, the cosmic curvature parameter will be constrained to ~ 0.057 or ~ 0.041 (1σ), depending on the availability of high-redshift supernovae, much more stringent than current model-independent techniques. We also show that the bias in the time delay method might be well controlled, leading to robust results. The proposed method is a new independent tool for both testing the fundamental assumptions of homogeneity and isotropy in cosmology and for determining cosmic curvature. It is complementary to cosmic microwave background plus baryon acoustic oscillation analyses, which normally assume a cosmological model with dark energy domination in the late-time universe.

Subject headings: gravitational lensing: strong — supernovae: general — methods: data analysis — distance scale

1. INTRODUCTION

Cosmology has been flourishing over the last decades with impressive improvements in the quality and quantity of astronomical observations. At the basis of most of the remarkable achievements in Cosmology lies the fundamental assumption that the universe on large scales is homogeneous and isotropic, as described by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric.

With a substantial amount of new data of unprecedented precision soon to become available, both from ongoing and forthcoming observational missions, we will be able to test the FLRW metric directly. The test should be independent of the matter contents in the universe and how they interact with spacetime geometry, for example, through the Einstein equation. Such a test will either strengthen our existing understanding or reveal new physics. Therefore, such tests deserve our full attention and it is important they can be performed once the new data become available. In fact, deviations from FLRW geometry have been proposed theoretically, and can provide an alternative explanation for the inferred late-time acceleration in our universe (Ferrer & Räsänen 2006; Enqvist 2008; February et. al. 2010; Bolejko et. al. 2011; Redlich et. al. 2014; Räsänen 2009; Boehm & Räsänen 2013).

Testing the FLRW metric was proposed in Clarkson et. al. (2008), where comparing observational determinations of the expansion rate and cosmological distances was suggested. The

method was applied in Shafieloo & Clarkson (2010); Mörtzell & Jönsson (2011); Sapone et. al. (2014). Another technique using parallax distance and angular diameter distance was proposed (Räsänen 2014). Further, Räsänen et. al. (2015) proposed that the observation of lensing systems, including separations of the images and central velocity dispersions, can be used in combination with supernova data to test the FLRW metric, in a model-independent way, through the Distance Sum Rule (DSR). They applied their method to the existing 23 lensing systems and found no violation. They also obtained a constraint of cosmic curvature: $-1.22 < \Omega_k < 0.63$ within 2σ uncertainty. Note that their method strongly depends on the universal lens models.

Strong lensing has been a powerful tool for both astrophysics and cosmology (Treu 2010). A typical system consists of a distant quasar, lensed by a foreground elliptical galaxy, forming multiple images of the AGN and producing time delays due to geometrical and the Shapiro effects. The time delay distance, which is a combination of three angular diameters can be extracted from the observed time delay light curves and high-resolution imaging from the space telescope. It contains information of the spacetime geometry and the properties of the matter/energy components in the universe. We find that the time delay distance is similar to the angular diameter distance ratio, which can be obtained under the assumption of a universal SIS model (and its extensions) of the lens (Räsänen et. al. 2015), suggesting the DSR works in this case as well. In the time delay method, however, this assumption is not employed.

We have therefore performed a detailed study based on the time delay distance. Note that the test consists of two levels: first, we test the FLRW metric through the DSR and then, if its validity is confirmed, we can obtain a constraint of cosmic curvature. There have been many works on constraining the curvature, mainly based on CMB and BAO, suggesting our universe is flat at the per cent level (Planck 2016; Eisenstein et. al. 2005; Tegmark et. al. 2006). However, these constraints are based on assuming specific cosmological models. Model-independent results have been given in Shafieloo & Clarkson (2010); Mörtzell & Jönsson (2011); Sapone et. al. (2014); Cai et. al. (2016), but the constraints were weak as these methods require constructing the derivative of noisy distance measure data. Recently, supernovae in combination with Hubble expansion data gave an improved constraint $\Omega_K = -0.140^{+0.161}_{-0.158}$ (Li et. al. 2016).

Throughout this paper, we take a flat Λ CDM universe with matter density $\Omega_M = 0.3$ and Hubble constant $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ as our fiducial model in simulations. The speed of light $c = 1$.

2. LENSING AND SUPERNOVA OBSERVATIONS

The current number of well-measured time delay lens systems is limited so that we cannot at present obtain an accurate enough test from existing data. Measuring time delays requires monitoring the light curves for quite a long time, usually years, with high cadence and long observing seasons. It also relies on accurate algorithms that can deal with the independent microlensing effects caused by star motions at different image environments. Once these conditions are met, one can get precise and accurate time delays, see for example the COSMOGRAIL program (Tewes et. al. 2013a,b).

It is inspiring that the upcoming Large Synoptic Survey Telescope (LSST) will find more than 8000 lensed quasars, some 3000 of which will have well-measured time delays (Oguri & Marshall 2010). A Time Delay Challenge (TDC) program has recently been initiated to test the accuracy of current algorithms (Liao et. al. 2015; Dobler et. al. 2015). The average precision of these time delay measurements has been shown to be $\sim 3\%$ through the first challenge (TDC1), comparable with current uncertainty of lens modelling (Suyu et. al. 2013) (see the HOLICOW program (Suyu et. al. 2016; Bonvin et. al. 2017)). Considering that the metric Efficiency defined in Liao et. al. (2015) is $\sim 20\%$, the TDC1 gave at least 400 well-measured time delay systems. Note that the TDC1 only simulated light curves in the i band and information on the images were not given, making the outlier problem severe. The efficiency is expected to be larger, $\sim 40\%$ in the LSST 6 band observation including images of the AGNs along with their hosts.

In Fig. 1 we show the redshift distributions of elliptical galaxies and quasars with well-measured time delays observed by LSST (Oguri & Marshall 2010). The source redshift is cut by 1.7, covering the supernova redshift range. There are more lensed quasars at higher redshifts, our test is limited by the maximum redshift of the supernovae observations. Considering the expected improvement of TDC results and the follow-up monitoring programs for existing and future low redshift source systems as complementary, as well as other projects like the

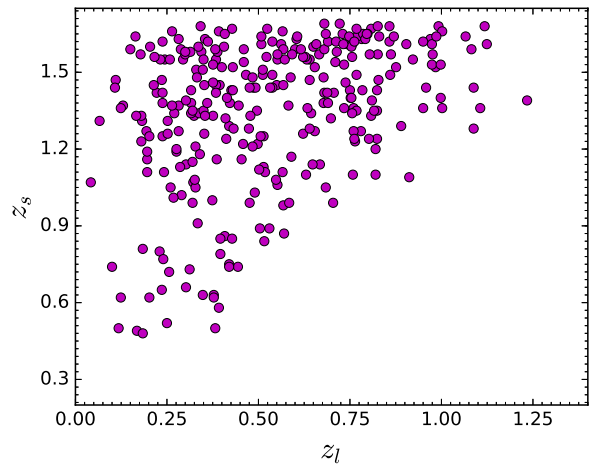


FIG. 1.— The lens and source redshift distributions of the lensing systems with well-measured time delay light curves observed by LSST ($z_s < 1.7$). The 40% efficiency of the state-of-the-art algorithms would result in ~ 300 systems.

Kunlun Dark Universe Telescope, the Dark Energy Survey (DES), the South Pole Telescope and so on, we take 100 lensed quasars with $z_s < 1.3$ as a benchmark, which at worst provides an order of magnitude estimate for testing the FLRW metric and determining cosmic curvature. This number is also consistent with Linder (2011). If the supernova observation can achieve $z \sim 1.7$, the corresponding number of lensed quasars would be up to ~ 300 .

The time delay is determined by both the mass distribution of the lens, and a combination of angular diameter distances known as the time delay distance:

$$D_{\Delta t} = \frac{D_A(z_l)D_A(z_s)}{D_A(z_l, z_s)}, \quad (1)$$

through

$$\Delta t = (1 + z_l)D_{\Delta t}\Delta\phi, \quad (2)$$

where z_l, z_s are redshifts at lens and source, respectively. $\Delta\phi$ is the Fermat potential difference between image positions, which can be inferred from high resolution imaging observations of the Einstein Ring due to the AGN host galaxy, combined with spectroscopic observations of the stellar kinematics of the lens galaxy. The uncertainty on $\Delta\phi$ is expected to be a few percent according to current techniques. Along with the uncertainty on time delay measurements, it gives $\sim 5\%$ uncertainty on the time delay distance, which is what was used in Linder (2011).

We consider supernova observations from DES which carries out a deep optical and near-infrared survey of 5000 deg^2 of the south Galactic cap using a new 3 deg^2 CCD camera mounted on the Blanco 4 m telescope at the Cerro Tololo Inter-American Observatory. It will provide a homogeneous sample of up to 4000 Type Ia supernovae to better study the nature of dark energy, though the prediction depends on the survey strategy. We take the simulation from the 10-field hybrid strategy, where the fields are the two deep fields and the three shallow fields from the 5-field hybrid strategy, plus additional shallow fields clustered around the Chandra Deep Field-South field. This strategy offers an attractive balance among all important considerations (Bernstein et. al. 2012). The

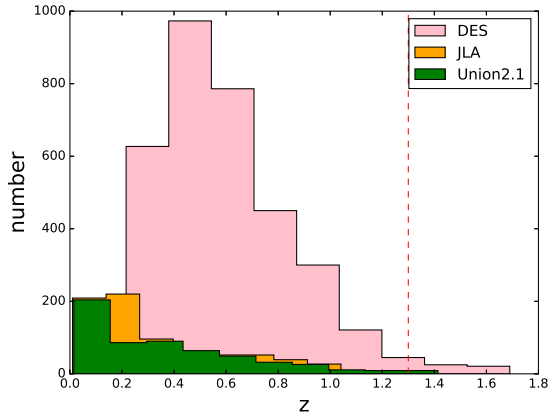


FIG. 2.— The redshift distributions of supernovae observed by future DES, as well as current Union2.1 and JLA samples. We extend the distribution to $z = 1.7$ according to the trend of DES.

redshift distribution is shown in Fig. 2 along with the current Union2.1 (Suzuki et. al. 2012) and JLA samples (Betoule et. al. 2014). We also expect relatively fewer high-redshift $\sim 1.3 < z < \sim 1.7$ supernovae will be found by future deep supernova surveys, though the concrete number is not known. To show the effects of high-redshift supernovae, we just follow the trend of DES prediction made by Bernstein et. al. (2012) and extend the redshift distribution to $z = 1.7$ in Fig. 2, there are ~ 60 high-redshift supernovae.

3. METHODOLOGY

The FLRW metric describes a homogeneous and isotropic universe:

$$ds^2 = -dt^2 + \frac{a(t)^2}{1 - Kr^2} dr^2 + a(t)^2 r^2 d\Omega^2, \quad (3)$$

where K is a constant describing spatial curvature. From this metric, the dimensionless distance $d(z_l, z_s)$ between two redshifts, which is related to the angular diameter distance $D_A(z_l, z_s)$ through $d(z_l, z_s) = (1 + z_s)H_0 D_A(z_l, z_s)$, is given by:

$$d(z_l, z_s) = \frac{1}{\sqrt{|k|}} \begin{cases} \sinh(\sqrt{-k}\chi) & k < 0 \\ \sin(\sqrt{k}\chi) & k > 0, \end{cases} \quad (4)$$

where we denote $k \equiv K/H_0^2 = -\Omega_K$ and $\chi = \int_{t_s(z_s)}^{t_l(z_l)} \frac{H_0 dt}{a(t)}$. Both of these expressions reduce to the flat case in the limit that k goes to zero. Through the Distance Duality Relation (Liao et. al. 2016), luminosity distance $D_L(z_l, z_s) = D_A(z_l, z_s)(1 + z_s)^2$.

Denoting $d(z) \equiv d(0, z)$ as in Räsänen et. al. 2015, the DSR that relates $d_{ls} \equiv d(z_l, z_s)$ to $d_l \equiv d(z_l)$ and $d_s \equiv d(z_s)$ can be written as:

$$d_{ls} = \epsilon_1 d_s \sqrt{1 - kd_l^2} - \epsilon_2 d_l \sqrt{1 - kd_s^2}, \quad (5)$$

where $\epsilon_i = \pm 1$, with $k \leq 0$ corresponding to $\epsilon_i = 1$. If $k > 0$, the signs rely on the locations of the lens and source at the three-dimensional hyper-sphere, as well as the direction of the light propagation. In this case, the FLRW metric covers only half of the spacetime. In

this work, we assume there exists a one-to-one correspondence between cosmic time t and redshift z , with $d'(z) > 0$; then $\epsilon_i = 1$. We rewrite the sum rule as

$$\frac{d_{ls}}{d_l d_s} = T(z_l) - T(z_s), \quad (6)$$

where

$$T(z) = \frac{1}{d(z)} \sqrt{1 - kd(z)^2}. \quad (7)$$

Different from Eq. 4 in Räsänen et. al. 2015, where the authors relate to the angular diameter distance ratio obtained from the observed velocity dispersion and image separation, the expression here relates to the observed time delay distance. Note that the sum rule becomes more symmetric on the right hand side.

To test the FLRW metric, one could use the consistency condition (Räsänen et. al. 2015) derived from Eq. 5:

$$k_S = -\frac{d_l^4 + d_s^4 + d_{ls}^4 - 2d_l^2 d_s^2 - 2d_l^2 d_{ls}^2 - 2d_s^2 d_{ls}^2}{4d_l^2 d_s^2 d_{ls}^2}, \quad (8)$$

where the subscript S stands for sum rule. In this k_S test, the $k_S(z_l, z_s)$ should be a constant equal to $-\Omega_{K0}$ if the FLRW metric is valid. Any two pairs that give different k_S would indicate a deviation from the FLRW metric – a violation of homogeneity and/or isotropy. However, the simulation suggests that the complexity of the expression may cause non-Gaussian effects which can lead to a bias. Also, the uncertainties are quite large for individual pairs. In order to get an unbiased estimation, one needs more exquisite statistics, and with current data quality, the k_S test seems impractical. A temporary strategy to testing the FLRW metric is to fit a constant k to the data (Räsänen et. al. 2015), in which case a large $\chi^2/d.o.f$ may indicate a violation of the FLRW metric assumption.

In this paper, we propose an effective and efficient evolution model to test the FLRW metric, where the violation criterion is specified concretely. As the split term T in Eq. 7 only relies on a certain redshift in the time delay method, we extend the constant k to be redshift-dependent $k(z)$:

$$T'(z) = \frac{1}{d(z)} \sqrt{1 - k(z)d(z)^2}, \quad (9)$$

where we simply parameterize $k(z)$ by a first-order Taylor expansion $k(z) = k_0 + k_E z$, the subscript E standing for evolution. The physical meaning of $k(z)$ deserves further study at the theoretical level. It could be related, for example, to the evolution of cosmic curvature (Godłowski et. al. 2004; Balcerzak et. al. 2015; Buchert et. al. 2009). In this work, we only focus on observationally testing the validity of the FLRW metric, rather than on the physical motivation of its alternatives. Therefore, the parameter k_E is used as the violation criterion. Any deviation of $k_E = 0$ is a sign of violation of the homogeneity and isotropy assumptions as represented by the FLRW metric ansatz.

On the second level of our test, if the data support the FLRW metric, we can obtain the probability distribution of k , i. e., we can constrain the value of the cosmic

curvature parameter, thereby testing the flatness of the spatial sections of the universe.

4. SIMULATIONS AND RESULTS

We simulated 100 and 300 lensed quasar-elliptical systems, with source redshifts below 1.3 and below 1.7 respectively from the OM10 catalogue (Oguri & Marshall 2010) that provides mock observations of LSST based on realistic distributions of quasars and elliptical galaxies, as well as the observing condition of the telescope. We considered a 3% uncertainty in time delay measurements and the same order uncertainty for lens modelling embodied as $\Delta\phi$. These would result in $\sim 5\%$ uncertainty in the time delay distance or its inverse for individual systems. For d_l and d_s , we simulated 3540 supernovae where 60 are $z > 1.3$ based on DES 10-field hybrid strategy.

We obtained $d(z)$ and $k(z)$ (or constant k) in model-independent way, through fitting to supernovae and lensing systems simultaneously. In particular, we parameterized $d(z)$ as a fourth order polynomial $d(z) = z + a_1 z^2 + a_2 z^3 + a_3 z^4$. Increasing the order does not affect the final result given the quality of the simulated data. Note that under the assumption that the high-redshift supernovae are much fewer, the form of the high-redshift extension in Fig. 2 affects little on determining $d(z)$. The biggest influence comes from the significantly increased number of lensed quasars.

We used the minimization function in Python to find the set of parameters that corresponds to minimum χ^2 . To get an unbiased estimation, we simulated 60000 realizations of the data with different random seeds and repeated the minimization process. Note that the cosmic distances rely on Hubble constant H_0 , we treated H_0 as a free parameter and marginalized over it like the coefficients of the polynomial. In principle, to achieve a model-independent result, for supernova observation, one has to take the original measurements of observed magnitude m , the stretch factor x and color parameter c in the distance modulus $\mu = m - M + \alpha x - \beta c$, where M is the absolute magnitude, α , β are nuisance parameters related to the well-known broader-brighter and blue-brighter relationships, respectively. (M, α, β) should be taken as free parameters. However, for our simulation, we only focus on the power of this method and ignore the detailed techniques. Therefore, we take the distance module as the observational quantities along with the uncertainties from Bernstein et. al. (2012).

The marginalized 2-D constraint contour and 1-D probability density distributions of k_0 and k_E are shown in Fig. 3. The marginalized distribution of k is shown in Fig. 4. These figures manifestly show that our time delay method is very powerful on both testing the FLRW metric assumption and constraining cosmic curvature. For cosmic curvature, the uncertainty is at least one order of magnitude smaller than that in Räsänen et. al. 2015. The proposed time delay method will also surpass the corresponding method using standard clocks (Li et. al. 2016) by at least three times. The numerical uncertainties are presented in Table. 1.

5. SUMMARY AND DISCUSSION

In this work we developed a new model-independent test of the FLRW metric and cosmic curvature using

z_{max}	k_0	k_E	k
1.3	0.271	0.161	0.057
1.7	0.157	0.078	0.041

TABLE 1
THE 1σ UNCERTAINTIES FOR THE PARAMETERS (k_0, k_E) IN FLRW TEST, AS WELL AS THE CURVATURE PARAMETER k . WE CONSIDER TWO CASES WHERE THE MAXIMUM REDSHIFTS OF SUPERNOVAE ARE 1.3 AND 1.7, RESPECTIVELY.

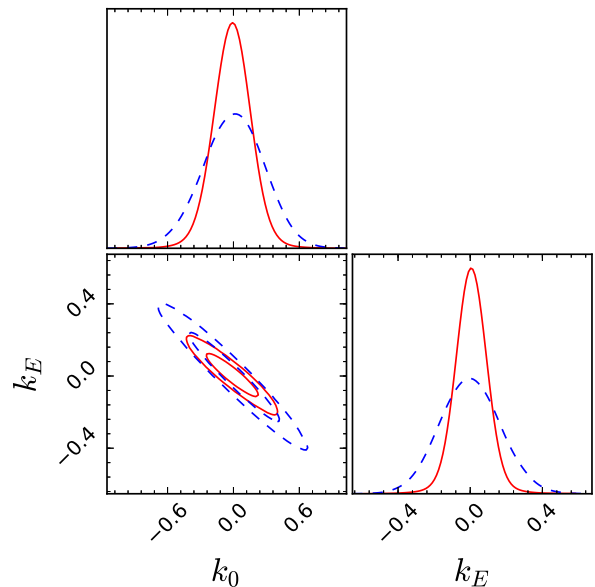


FIG. 3.— The one-dimensional marginalized distributions and 68%, 95% confidence contours from 60000 minimization points for the parameters in FLRW test. Two cases are considered where the maximum redshifts of supernovae are 1.3 (blue line) and 1.7 (red line), respectively.

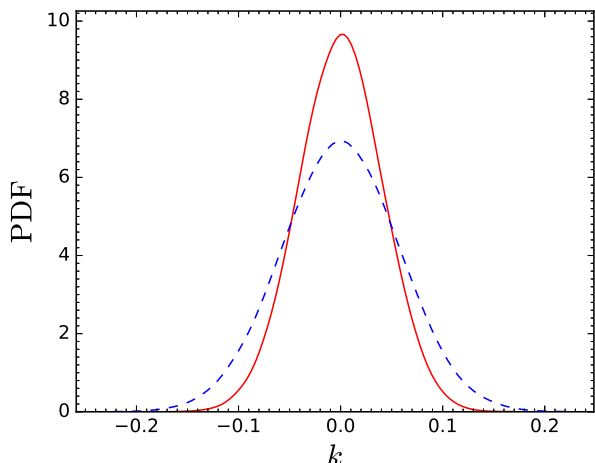


FIG. 4.— The marginalized distributions of k from 60000 realization minimizations. The colors are consistent with Fig. 3.

strong lens time delay systems and supernovae, based on the Distance Sum Rule. We also introduced an evolution model where the violation criterion of the FLRW metric is specified. Our method should not be simply deemed as an extension of Räsänen et. al. 2015, where the au-

thors used the angular diameter distance ratio derived from lens velocity dispersions and image separations.

The distance ratios were derived under the assumption that there exists a simple universal lens model for all lenses, like, for example, the SIS model or its extensions. In fact, several studies have shown that the universal SIS model (or its extensions) brings large $\chi^2/d.o.f.$ values, usually $\sim 3-4$ in cosmology (Xia et. al. 2017; Cao et. al. 2015, 2012). The reason could be that the environments of individual systems may be quite different, e.g. due to the effects of nearby galaxies, the densities of galaxies along the line of sight, or the variation of the lens slopes. If these can be seen as systematic errors that only enlarge the uncertainty rather than bias the estimation, one may use the D’Agostini’s likelihood (Xia et. al. 2017) where a constant intrinsic scatter σ_{int} , representing any other unknown uncertainties except for the observational statistical ones, is introduced. The other way is to simply add an extra Gaussian systematic error to the distance ratio, or the parameter f_e that characterizes the difference between the velocity dispersion of the observed stars and the SIS model velocity dispersion and accounts for other systematic errors (Kochanek et. al. 2000). However, there might be another possibility, namely that the large $\chi^2/d.o.f.$ is not caused by systematic errors, but is a bias by the model itself, i. e. assuming a simple universal model for all lenses is invalid. Outliers are easy to occur if one does not take individuals into consideration. The dispersion and separation observations may not be sufficient to remove the outliers. In any case, the current distance ratio technique requires a detailed bias investigation, otherwise results may not be reliable.

The time delay method, on the other hand, is expected to become an unbiased technique. It focuses on individual lensing systems, each lens can be given an independent flexible model. The measurements of time delays and the observation of the AGN images along with their host can produce strong constraints on these individual lens models. There have been “blind analyses” conducted to control the bias and make the accuracy far smaller than the precision for both time delay measurements (Liao et. al. 2015) and lens modelling (Suyu et. al. 2016; Wong et. al. 2016). We emphasize that although lens potentials for time delay lenses have been determined accurately, it is currently hard to predict whether there is any residual systematics after combining many of such time delay lenses. Therefore, the lens model uncertainty is still very important issue that we have to take seriously. Furthermore, the distance ratio relies on σ^2 , the square giving rise to a large uncertainty, while the time delay method relies on the first power for both Δt and $\Delta\phi$.

The simulation shows that our method can test the FLRW metric effectively and efficiently with an uncertainty for $k_E \sim 0.078 - 0.161$. Under the validity of

the FLRW metric, one could achieve an uncertainty on the cosmic curvature parameter k of $\sim 0.041 - 0.057$ in model-independent way, but quite accurately; indeed much more precise than all current techniques. Therefore, we do not need priors from the Hubble constant and the CMB like in Räsänen et. al. 2015. The independent knowledge of cosmic curvature will significantly contribute to a better understanding of the evolution history of the universe and of the nature of dark energy as well as inflationary theory which strongly favours a flat universe. The time delay method proposed here does not involve taking derivatives of distance measurements (Shafieloo & Clarkson 2010; Mörtzell & Jönsson 2011; Sapone et. al. 2014), which introduces large uncertainties. The observational data required for the test will be acquired in the very near future, while the angular diameter and parallax distance method may suffer from a lack of data (Räsänen 2014).

6. PERSPECTIVES

According to the investigation of the LSST observing strategy, a limitation is the relatively low redshifts of supernovae, compared to a typical redshift $\sim 2 - 3$ of the lensed quasars observed in LSST. To enhance this method, one could also utilize more high-redshift luminosity distance sources like gamma ray bursts, quasars (Risaliti & Lusso 2015) or even gravitational wave observations from standard sirens. On the other hand, the follow-up monitoring programs and the $1 - 2\ m$ dedicated telescopes applied in existing systems where the source redshifts are comparable with supernovae are encouraged. With more useful lensed quasar systems, the test would be increasingly improved.

In addition, more kinds of lensing systems are promising in the future, for example, the lensed supernova whose maximum intensity of the light curve could make contribution to the time delay measurement. The recent detection of gravitational waves has opened a new window for astrophysics and cosmology. The time delay of lensed GW with its counterpart, like kilonova/mergenova, short gamma ray burst or fast radio burst, can be measured quite accurately due to the characteristic waveform, which may also benefit the proposed test (Fan et. al. 2017).

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